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# Descent Methods for Equilibrium Problems in a Banach Space

O. CHADLI
Department of Economics, Ibn Zohr University
B.P. 8658 Hay Dakla, Agadir, Morocco

I. V. KONNOV
Department of Applied Mathematics
Kazan State University
Kazan, Russia

J. C. YAO
Department of Applied Mathematics
National Sun Yat-Sen University
Kaohsiung, Taiwan, R.O.C.

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**Abstract**—In this paper, we consider equilibrium problems with differentiable bifunctions in a Banach space setting and investigate properties of gap functions for such problems. We suggest a derivative-free descent method and give conditions which provide strong convergence of the method.


**Keywords**—Equilibrium problems, Differentiable bifunctions, Gap function, Feasible descent method.

## 1. INTRODUCTION

Let $U$ be a nonempty closed convex subset of a reflexive Banach space $E$ and $h : U \times U \to R$ an equilibrium bifunction, i.e., $h(u,u) = 0$ for every $u \in U$. Then, one can define the *equilibrium problem* (EP) that is to find an element $\bar{u} \in U$ such that

$$h(\bar{u}, v) \geq 0, \qquad \forall u \in U. \tag{1}$$

This problem was investigated by many researchers, both in finite- and infinite-dimensional spaces; e.g., see [1–4] and references therein. By introducing a gap function, one can reduce EP (1) to a scalar optimization problem; see [2,5]. At the same time, in order to find a solution to EP (1) by solving the corresponding optimization problem, it is necessary for this problem

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